GEOMETRIC SERIES

$(x \in \mathbb{R})$
Let $|x| < 1$. Suppose

$$S = 1 + x + x^2 + \dots + x^n \qquad (n \in \mathbb{N})$$

$$\Rightarrow xS = \quad x + x^2 + \dots + x^n + x^{n+1}$$

Subtracting,

$$S(1-x) = 1 - x^{n+1} \qquad \Rightarrow S = \frac{1 - x^{n+1}}{1-x}.$$

If $n \to \infty$, $x^{n+1} \to 0$ since $|x| < 1$.

So, $$1 + x + x^2 + \dots = \sum_{i=0}^{\infty} x^i = \frac{1}{1-x}.$$

Such an ~~series~~ expression is said to be in geometric progression with factor x and the infinite sum $1 + x + x^2 + \dots$ is called a geometric series, that is only convergent when $|x| < 1$.