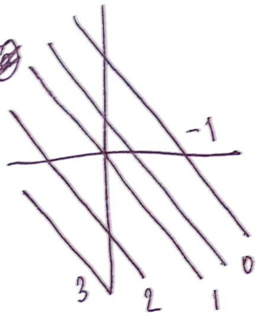
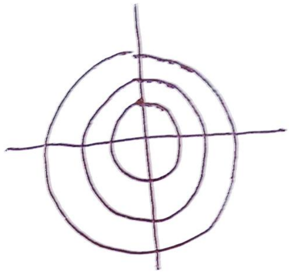
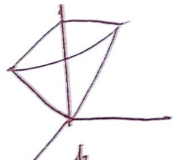
1



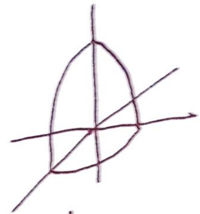


(a)

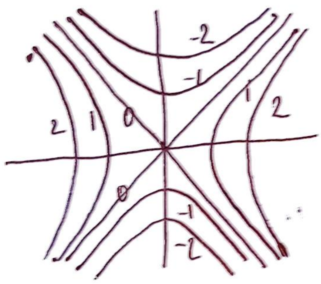
b, c, d



b



d



c

2. (a) $f_x = 3x^2y - 3y^2$, $f_y = x^3 - 6xy + 4y$

(b) $z_x = \frac{1}{y}$, $z_y = -\frac{x}{y^2}$

(c) $f_x = 3\cos(3x + 2y)$, $f_y = 2\cos(3x+2y)$

(d) $f_x = 2xy\, e^{x^2y}$, $f_y = x^2 e^{x^2y}$

(e) $z_x = \ln(2x+y) + \frac{2x}{2x+y}$, $z_y = \frac{x}{2x+y}$

(f) $f_x = 2xz$, $f_y = -2z^3$, $f_z = x^2 - 6yz^2$.

3 (a) both sides are $mnx^{m-1}y^{n-1}$

(b) $f_x = \frac{y}{(x+y)^2}$, $f_{xy} = (f_x)_y = (f_x)_y = \frac{x-y}{(x+y)^3}$; $f_y = \frac{-x}{(x+y)^2}$, $f_{yx} = \frac{-(y-x)}{(x+y)^3}$

(c) $f_x = -2x \sin(x^2+y)$, $f_{xy} = (f_x)_y = -2x \cos(x^2+y)$

$f_y = -\sin(x^2+y)$, $f_{yx} = -\cos(x^2+y) \cdot 2x$

(d) both sides are $f'(x)\, g'(y)$

4. $(f_x)_y = ax + 6y$, $(f_y)_x = 2x + 6y$; therefore $f_{xy} = f_{yx} \Leftrightarrow a = 2$. By inspection, one sees that if $a=2$, $f(x,y) = x^2 y + 3xy^2$ is a function with the given f_x and f_y.

5. (a) $z_x = y^2$, $z_y = 2xy$; therefore at $(1,1,1)$ we get $z_x = 1$, $z_y = 2$ so that the tangent plane is $z = 1 + (x-1) + 2(y-1)$ or $z = x + 2y - 2$.

(b) $w_x = -\dfrac{y^2}{x^2}$, $w_y = \dfrac{2y}{x}$; therefore at $(1,2,4)$, we get $w_x = -4$, $w_y = 4$, so that the tangent plane is $w = 4 - 4(x-1) + 4(y-2)$, or $w = -4x + 4y$.

6. $z_x = \dfrac{x}{\sqrt{x^2+y^2}} = \dfrac{x}{z}$; by symmetry (interchanging x and y), $z_y = \dfrac{y}{z}$;

then the tangent plane is $z = z_0 + \dfrac{x_0}{z_0}(x - x_0) + \dfrac{y_0}{z_0}(y - z_0)$, or

$z = \dfrac{x_0}{z_0} x + \dfrac{y_0}{z_0} y$ since $x_0^2 + y_0^2 = z_0^2$

7. (a) $dw = \dfrac{dx}{x} + \dfrac{dy}{y} + \dfrac{dz}{z}$

(b) $dw = 3x^2y^2z\, dx + 2x^3yz\, dy + x^3y^2\, dx$

(c) $dz = \dfrac{2y\, dx - 2x\, dy}{(x+y)^2}$

(d) $dw = \dfrac{t\, du - u\, dt}{t\sqrt{t^2-u^2}}$

8. (a) $\frac{dw}{w^2} = \frac{dt}{t^2} + \frac{du}{u^2} + \frac{dv}{v^2}$; $\therefore dw = w^2\left(\frac{dt}{t^2} + \frac{du}{u^2} + \frac{dv}{v^2}\right)$

(b) $2u\, du + 4v\, dv + 6w\, dw = 0 \Rightarrow dw = -\frac{u\,du + 2v\,dv}{3w}$

9 (a) $\nabla f = 3x^2 i + 6y^2 j$; $(\nabla f)_P = 3i + 6j$; $\left.\frac{df}{ds}\right|_u = (3i+6j)\cdot\frac{i-j}{\sqrt{2}}$

$= -\frac{3\sqrt{2}}{2}$

(c) $\nabla f = 2(u+2v+3w)\,(i+2j+3k)$

$(\nabla f)_P = 4\,(i+2j+3k)$

$\left.\frac{df}{ds}\right|_u = 4\,(i+2j+3k)\cdot\frac{-2i+2j\,k}{3} = -\frac{4}{3}$.

10. (a) $\nabla f = \langle y^2z^3, 2xyz^3, 3xy^2z^2\rangle$; $(\nabla f)_P = \langle 4, 12, 36\rangle$

normal at P : $\langle 1, 3, 9\rangle$

tangent plane at P : $x + 3y + 9z = 18$

(c) $(\nabla w)_P = \langle 2x_0, 2y_0, -2z_0\rangle$; tangent plane : $x_0(x-x_0) + y_0(y-y_0) - z_0(z-z_0) = 0$

or $x_0x + y_0y - z_0z = 0$ since $x_0^2 + y_0^2 - z_0^2 = 0$.

11. (a) (i) $\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt} + \frac{\partial w}{\partial z}\frac{dz}{dt} = yz\cdot 1 + xz\cdot 2t + xy\cdot 3t^2 = t^5 + 2t^5 + 3t^5$

$= 6t^5$.

(ii) $w = xyz = t^6$; $\frac{dw}{dt} = 6t^5$.

13 In these, denote by $D = x^2 + y^2 + z^2$ the square of the distance from the point (x, y, z) to the origin, then the point which minimizes D will also minimize the actual distance.

(a) Since $z^2 = \frac{1}{xy}$, we get on substituting $D = x^2 + y^2 + \frac{1}{xy}$ with x and y independent; setting the partial derivatives equal to zero, we get

$$D_x = 2x - \frac{1}{x^2 y} = 0, \quad D_y = 2y - \frac{1}{y^2 x} = 0 ;$$

or $$2x^2 = \frac{1}{xy}, \quad 2y^2 = \frac{1}{xy}.$$

$$\Rightarrow x^2 = \frac{1}{2xy} = y^2$$ from which we get $y = \pm x$.

If $y = x$, then $x^4 = \frac{1}{2}$ and $x = y = 2^{-\frac{1}{4}}$ and so $z = 2^{1/4}$;

if $y = -x$, then $x^4 = -\frac{1}{2}$ and there are no solutions.

Thus the unique point is $\left(\frac{1}{2^{1/4}}, \frac{1}{2^{1/4}}, 2^{1/4}\right)$.

(b) Using the relation $x^2 = 1 + yz$ to eliminate x, we have $D = 1 + yz + y^2 + z^2$ with y and z independent; setting the partial derivatives equal to zero we get

$$D_y = 2y + z = 0, \quad D_z = 2z + y = 0.$$

$\hookrightarrow$ $y = z = 0 \Rightarrow x = \pm 1$

$\Rightarrow$ there are 2 points $(\pm 1, 0, 0)$ both at distance 1 from the origin.