① Sketch five level curves for each of the following functions. Also for a–d sketch the portion of the graph of the function lying in the first octant; include in your sketch the traces of the graph in the 3 coordinate planes, if possible.

ⓐ $1-x-y$ ⓑ $\sqrt{x^2+y^2}$ ⓒ x^2+y^2 ⓓ $1-x^2-y^2$ ⓔ x^2-y^2

② Calculate the first partial derivatives of each of the following functions:

ⓐ $w=x^3y-3xy^2+2y^2$ ⓑ $z=\frac{x}{y}$ ⓒ $\sin(3x+2y)$ ⓓ e^{x^2y}

ⓔ $z=x\ln(2x+y)$ ⓕ x^2z-2yz^3

③ Verify that $f_{xy}=f_{yx}$ for each of the following:

ⓐ $x^m y^n$ $(m,n\in\mathbb{Z}_+)$

ⓑ $\frac{x}{x+y}$

ⓓ $f(x)\,g(y)$ for any differentiable f and g.

ⓒ $\cos(x^2+y)$

④ By using $f_{xy}=f_{yx}$, tell for what value of the ~~[illegible]~~ constant a, there exists a function $f(x,y)$ for which $f_x=axy+3y^2$, $f_y=x^2+6xy$, and then using this value, find such a function by inspection.

⑤ Give the equation of the tangent plane to each of the surfaces at the point indicated.

ⓐ $z=xy^2$, $(1,1,1)$ ⓑ $w=y^2/x$ $(1,2,4)$

6. Find the equation of the tangent plane to the cone $z = \sqrt{x^2+y^2}$ at the point $P_0 : (x_0, y_0, z_0)$ on the cone.

7. Find the differential :

ⓐ $w = \ln(xyz)$; ⓑ $w = x^3 y^2 z$; ⓒ $z = \dfrac{x-y}{x+y}$; ⓓ $w = \sin^{-1}\dfrac{u}{t}$

[use $\sqrt{t^2-u^2}$]

8. The following equations define w implicitly as a function of the other variables. Find dw in terms of all the variables by taking the differential of both sides and solving algebraically for dw.

ⓐ $\dfrac{1}{w} = \dfrac{1}{t} + \dfrac{1}{u} + \dfrac{1}{v}$; ⓑ $u^2 + 2v^2 + 3w^2 = 10$.

9. In each of these, a function f, a point P and a vector A are given. Calculate the gradient of f at the point, and the directional derivative $\left.\dfrac{\partial f}{\partial s}\right|_u$ at the point in the direction u of the given vector A.

ⓐ $x^3 + 2y^3$; $(1,1)$; $i - j$

ⓑ $w = \dfrac{xy}{z}$; $(2,-1,1)$, $i + 2j - 2k$

ⓒ $z = x\sin y + y\cos x$; $(0, \frac{\pi}{2})$, $-3i + 4j$

ⓓ $w = \ln(2t + 3u)$; $(-1, 1)$, $4i - 3j$

ⓔ $f(u,v,w) = (u + 2v + 3w)^2$; $(1,-1,1)$, $-2i + 2j - k$.

10. By viewing the following surfaces as a contour surface of a function $f(x,y,z)$, find its tangent plane at the given point.

ⓐ $xy^2z^3 = 12$, $(3, 2, 1)$

(b) the ellipsoid $x^2+4y^2+9z^2=14$; $(1,1,1)$

(c) the cone $x^2+y^2-z^2=0$, (x_0, y_0, z_0)

11. Find $\frac{df}{dt}$ for the composite function $f(x(t), y(t), z(t))$ in two ways:

(i) use chain rule, then express your answer in terms of t by using $x=x(t)$ etc

(ii) express the composite function f in terms of t and differentiate.

(a) $w = xyz$ $\left(x=t,\ y=t^2,\ z=t^3\right)$

(b) $w=x^2-y^2$, $x=\cos t$, $y=\sin t$

(c) $w=\ln(u^2+v^2)$, $u=2\cos t$, $v=2\sin t$

12. (a) Let $w=f\left(\frac{y}{x}\right)$. Show that w satisfies the PDE $x\frac{\partial w}{\partial x}+y\frac{\partial w}{\partial y}=0$

(b) $w=f(x^2-y^2)$ $\longrightarrow$ $y\frac{\partial w}{\partial x}+x\frac{\partial w}{\partial y}=0$

(c) $w=f(ax+by)$ $\longrightarrow$ $b\frac{\partial w}{\partial x}-a\frac{\partial w}{\partial y}=0$.

13. Find the point(s) on these surfaces which are closest to the origin

(a) $xyz^2=1$

(b) $x^2-yz=1$

Hint: It is easier to minimise the square of the distance, rather than the distance itself.