Tutorial Sheet on Functions

1)

$$|s_{n+1}-s_n| = |f(s_n)-f(s_{n-1})|$$

$$\Rightarrow \frac{|s_{n+1}-s_n|}{|s_n-s_{n-1}|} = \frac{|f(s_n)-f(s_{n-1})|}{|s_n-s_{n-1}|} = f'(c)$$ by mean value theorem where $c \in [s_n, s_{n-1}]$ or $[s_{n-1}, s_n]$

Let $a = \sup |f'(x)| < 1$

Then,

$$\frac{|s_{n+1}-s_n|}{|s_n-s_{n-1}|} = f'(c) \leq a$$

$$\Rightarrow 0 \leq |s_{n+1}-s_n| \leq a\,|s_n-s_{n-1}| \quad \forall n \in \mathbb{N}.$$

$\therefore \{s_n\}_{n\in\mathbb{N}}$ is Cauchy and hence convergent.